

PROXIMAL SPLITTING ALGORITHMS IN NONLINEAR SPACES

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Abstract

In the setting of $\text{CAT}(\kappa)$ spaces, common fixed point iterations built from prox mappings (e.g. prox-prox, Krasnoselsky–Mann relaxations, nonlinear projected- gradients) converge locally linearly under the assumption of linear metric subregularity. Linear metric subregularity is in any case necessary for linearly convergent fixed point sequences, so the result is tight. To show this, we develop a theory of fixed point mappings that violate the usual assumptions of nonexpansiveness and firm nonexpansiveness in p -uniformly convex spaces.

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1 References

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